Hysteretic Dynamic Effect in Unsaturated Soil

S. Majid Hassanizadeh

Department of Earth Sciences
Utrecht University
The Netherlands

Collaborators:

ZHUAU NG Luwen; Utrecht University, The Netherlands

Susanne Fritz; Stuttgart University, Germany

Simona Bottero; Delft University of Technology, The Neth.

Rainer Helmig; Stuttgart University, Germany

Rien van Genuchten; Utrecht University, The Netherlands

Michael Celia; Princeton University, USA
Outline

Introduction

What is dynamic effect?

Experimental evidences

Dynamic capillarity model

Modelling experiments with standard model and dynamic capillarity model

Conclusions
Standard two-phase flow equations

\[ n \frac{\partial S^\alpha}{\partial t} + \nabla \cdot \mathbf{q}^\alpha = 0 \]

\[ \mathbf{q} = -\frac{K}{\mu}(\nabla P - \rho g) \]
Standard two-phase flow equations

\[ n \frac{\partial S^\alpha}{\partial t} + \nabla \cdot q^\alpha = 0 \]

\[ q^\alpha = - \frac{K^\alpha}{\mu^\alpha} \left( \nabla P^\alpha - \rho^\alpha g \right) \]
Standard two-phase flow equations

\[ n \frac{\partial S^\alpha}{\partial t} + \nabla \cdot q^\alpha = 0 \]

\[ q^\alpha = \frac{k^{r\alpha}}{\mu^\alpha} \mathbf{K} \cdot \left( \nabla P^\alpha - \rho^\alpha \mathbf{g} \right) \]

\[ k^{r\alpha} = k^{r\alpha} \left( S^w \right) \]

\[ P^n - P^w = f \left( S^w \right) = P^c \]
Measurement of Capillary Pressure-Saturation Curve in a pressure plate

\[ P^n_{\text{ext}} - P^w_{\text{ext}} = P^c_{\text{int}} = f(S^w) \]

In the case of soils, often it takes more than one week to get a set of wetting and drying curves.
There is no unique $P_c$-S curve.
Experiments on the uniqueness of $P_c$-$S$ curves

Topp, Klute, and Peters, 1967

These are all primary drainage curves
Experiments on the uniqueness of $P^c - S$ curves

Smiles, Vachaud, and Vauclin, 1971

Equilibrium imbition

Nonequilibrium imbition

Equilibrium drainage

Nonequilibrium drainage

Water content $\theta$, cm$^3$/cm$^3$
Two-phase flow dynamic experiments (PCE and Water)

Selective pressure transducers used to measure average pressure of each phase within the porous medium.

Hassanizadeh, et al., 2004
Two-phase flow dynamic experiments (PCE and Water)

(Static) $P^c$-S Curves

- Primary drainage
- Main drainage
- Main imbibition

$P_n - P_w$ in kPa

Saturation, $S$

Hassanizadeh, Oung, and Manthey, 2004
Two-phase flow dynamic experiments (PCE and Water)

- Prim drain PN~16kPa
- Prim drain PN~20kPa
- Prim drain PN~25kPa
- Main imb PW~0kPa
- Main imb PW~5kPa
- Main imb PW~8kPa
- Main imb PW~10kPa
- Main imb PW~0kPa last
- Main drain PN~16kPa
- Main drain PN~20kPa
- Main drain PN~25kPa
- Main drain PN~30kPa
- Static Pc inside

Saturation, S

Prim. drain PN~16kPa
Prim. drain PN~20kPa
Prim. drain PN~25kPa
Main imb PW~0kPa
Main imb PW~5kPa
Main imb PW~8kPa
Main imb PW~10kPa
Main imb PW~0kPa last
Main drain PN~16kPa
Main drain PN~20kPa
Main drain PN~25kPa
Main drain PN~30kPa
Static Pc inside

Hassanizadeh, Oung, and Manthey, 2004
Experiments on the uniqueness of $P^c - S$ curves

Das and Mirzaee, AIChE, 2012
Unsaturated flow in industrial porous media

Penetration of a liquid into diaper
Measurement of Pc-S curves for layers of a diaper

- Membrane is fully saturated during measurement.
- During drainage, sample is initially fully wet and gas pressure is zero; gas pressure is increased in steps.
- During imbibition, sample is initially dry and gas pressure is very high; gas pressure is decreased in steps.
Measurement of Pc-S curves for layers of a diaper

Pc-S curves for a single layer, and stacks of 3 layers or 10 layers
Each set of curves takes 12 hours to obtain!
Dynamic desaturation of a layer of diaper

![Graph showing dynamic desaturation of a layer of diaper with three simulations and one experiment, each labeled with parameters n=4 and tau=0, 5000, 10000 respectively.](image-url)
Multistep Outflow Experiments

O’CARROLL ET AL.: EXPLORING DYNAMIC EFFECTS IN CAPILLARY PRESSURE

Water volume (mg)

Time (hr)

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Experimental Set-up for measurement of dynamic capillarity effect

- No membranes
- Steady-state flow of invading fluid with incremental pressure increase
- Primary drainage
- Main drainage
- Main imbibition
- Transient drainage with large injection pressure

Bottero et al., WRR, 2011
Non-monotonic change of pressure difference with time

Bottero, 2009
Vertical infiltration of water in (almost) dry soil

Experiments by Rezanejad, 2002
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Infiltration fingers during penetration of water into (almost) dry soil

Infiltration experiments;
Rezanejad, 2002
Vertical infiltration of water in (almost) dry soil

Experiments by Rezanejad, 2002
Standard theory does not model non-monotonic distribution of saturation in infiltration fingers

Di Carlo, WRR. 2004
Non-monotonic distribution of saturation during infiltration into dry soil; experiments in our gamma system
Non-monotonic distribution of saturation during infiltration into dry soil; experiments in our gamma system

At $z=20$ cm and for different flow rates $q$ (in cm/min)

Fritz et al., 2012
Non-monotonic distribution of pressure during infiltration into dry soil; experiments in our gamma system

Pressure at different positions along the column

Fritz et al., 2012
Non-monotonic distribution of saturation during infiltration into dry soil; experiments in our gamma system

![Graph showing non-monotonic distribution of saturation.](image)
Shortcomings of the standard two-phase flow equations

Capillary-saturation curves are measured during a very slow procedure and are used to model fast flow processes.

We get different ‘capillary pressure’-saturation curves when a fast procedure is followed.

In simulations, change of saturation with time is always faster than in measurements.

Observed non-monotonic saturation distribution cannot be modelled.

Observed fingering during vertical infiltration of water into dry soil cannot be modelled.
From Averaging-Thermodynamic Approach, we derive:

\[ P^n - P^w = P_c \left( S^w \right) - \tau \frac{\partial S^w}{\partial t} \]

Hassanizadeh and Gray (WRR, 1993)
Two-phase flow dynamic experiments (PCE and Water)

\[ P^n - P^w = P_c(S) \]
Non-standard two-phase flow equations

\[ n \frac{\partial S^\alpha}{\partial t} + \nabla \cdot q^\alpha = 0 \]

\[ q^\alpha = -\frac{k_r^{\alpha\alpha}}{\mu^\alpha} \mathbf{K} \cdot (\nabla P^\alpha - \rho^\alpha \mathbf{g}) \]

\[ k_r^{\alpha\alpha} = k_r^{\alpha\alpha} \left( S^w \right) \]

\[ P^n - P^w = P_c(S) - \tau \frac{\partial S}{\partial t} \]

These equations are conditionally unstable and may have a non-monotonic solution.
Modified Richards Equation

One dimensional:

\[ n \frac{\partial S^w}{\partial t} = \frac{\partial}{\partial x} \left[ K^w \left( \left| \frac{dP^c}{dS^w} \right| \frac{\partial S^w}{\partial x} - 1 + \tau \frac{\partial^2 S^w}{\partial x \partial t} \right) \right] \]

These equations are conditionally unstable and may have a non-monotonic solution.
Simulating Multistep Outflow Experiments

O’CARROLL ET AL.: EXPLORING DYNAMIC EFFECTS IN CAPILLARY PRESSURE

Water volume (mg)

Fluid Phase Boundary Pressure (cm H2O)

Time (hr)

OBSERVED WATER OUTFLOW
3 PARAMETER FIT (Swr, VGB alpha & n)
3 PARAMETER FIT (Swr, BCM Pd & lambda)
WATER PHASE BOUNDARY PRESSURE AT BOTTOM OF COLUMN
PCE PHASE BOUNDARY PRESSURE AT TOP OF COLUMN

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Simulating Multistep Outflow Experiments

- Experimental Outflow
- 4 parameter fit (Swr, VGB alpha & n, constant tau)
- 4 parameter fit (Swr, VG alpha & n, tau a linear function of Sw)

Water volume (mg) vs. Time (hr)
Measurement of Pc-S curves for layers of a diaper

Diagram:
- Gas
- Membrane
- Glass frit
- Sample
- Balance

Samples: $P_1$, $P_2$
Simulating dynamic desaturation of a layer of diaper
Simulation of non-equilibrium primary drainage;

![Graph showing non-equilibrium primary drainage](image-url)
Non-monotonic distribution of saturation during infiltration into dry soil; experiments in our gamma system
Simulation of non-monotonic distribution of saturation during infiltration into dry soil
Modified Richards Equation

One dimensional:
\[
n \frac{\partial S^w}{\partial t} = \frac{\partial}{\partial x} \left[ K^w \left( \left| \frac{dP^c}{dS^w} \right| \frac{\partial S^w}{\partial x} - 1 + \tau \frac{\partial^2 S^w}{\partial x \partial t} \right) \right]
\]

Two dimensional:
\[
n \frac{\partial S^w}{\partial t} = \nabla \cdot \left[ K \cdot \left( \left| \frac{dP^c}{dS^w} \right| \nabla S^w - \rho^\alpha \mathbf{g} + \tau \nabla \frac{\partial S^w}{\partial t} \right) \right]
\]

These equations are conditionally unstable and may have a non-monotonic solution.
Development of vertical wetting fingers in dry soil; Simulations based on new capillarity theory
New experimental setup for study hysteretic dynamic effect

Sand cell dimensions: 2x3x3cm

Balance and transducers reading: every 0.5 s
Duration: around 5 h

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New experimental setup for study hysteretic dynamic effect
Dynamic drainage

Boundary suction: -50 cm
New experimental setup for study hysteretic dynamic effect

Dynamic drainage

Boundary suction: -70 cm
Conclusions

Standard unsaturated theory (with or without hysteresis) does not model nonmonotonic saturation distribution observed in capillary fingers.

Dynamic capillarity model does reproduce nonmonotonic saturation distribution.

Dynamic capillarity allows for modelling unstable fingers in 2-D or 3-D experiments.
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